

INERTIAL-CAPILLARY SURFACE WAVES AND THEIR INFLUENCE ON CRYSTAL GROWTH IN ZERO GRAVITY

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The results of the numerical calculations of oscillations of the liquid column free surface under the action on it of axial vibration in zero gravity are presented. On the basis of these calculations, an analytical model of the surface standing wave named the inertial-capillary wave has been developed. Comparison of the analytical calculations with the data of the numerical calculations and the experiment performed in microgravity under the TEXUS program has been made. The numerical study of the thermocapillary convection stability in the presence of inertial-capillary waves has shown that the change to the oscillation regime is very sharp upon reaching certain values of the vibration frequency and amplitude. The heat-and-mass transfer in growing semiconductor crystals by the floating-zone method in zero gravity under the action of vibration with allowance for the surface waves has been investigated. The possibility of measuring vibrational accelerations on-board space vehicles by means of oscillations of the liquid-column free surface are discussed.

Introduction. The behavior of a liquid with free boundaries in zero gravity has been a topic of enhanced interest since the beginning of spaceflight. This is due to not only the unusual form of fluid volumes and the specific feature of their motion in zero gravity, but also their important technical applications. The problem of prolonged storage of liquids, especially low-boiling ones, the motion and removal of gas inclusions from the liquid, the crystallization of a liquid having a free boundary — these are questions that cannot be resolved on the basis of the experience gained in hydromechanics. Of particular interest are dynamic processes, in particular, those connected with the oscillations of free surfaces under the action of the rotary motion of spacecraft and vibrations existing on board.

The waves on the free surface of a liquid filling a part of a cylindrical volume were investigated by the numerical simulation method in [1, 2] under the action of uniform residual accelerations of gravity, rotation, and a low-frequency vibration. The spectra of the surface waves were obtained. The behavior of the free surface of liquid hydrogen in a cylindrical vessel at different evolutions of the gravitational vector was considered in [3]. The calculations revealed the effect of a sharp burst on the free surface (geyser effect). These works lack generalization of the results obtained.

The behavior of the free surface of a fluid cylindrical volume with solid faces under the action of axial vibration in practically zero gravity was investigated in more detail in [4–9]. This problem was posed in connection with works on growing monocrystals by the floating-zone method onboard spacecraft. The analytical models of vibration-induced free surface oscillations [4–6] assumed that the standing wave consisted of one period and oscillated with the applied vibration frequency and that as soon as the vibration frequency reached the natural frequency of the liquid column, a sharp, resonance increase in the standing-wave amplitude occurred. Experiments in microgravity were performed on sounding rockets (TEXUS project), on orbital spacecraft (projects SL-1, SL-D1, and SL-D2) [7, 8], and on a release tower [9]. However, the experiments in [7, 8] were performed in a narrow range of influencing parameters, which made it impossible to reliably verify these analytical models. In [9], it is noted that no resonance increase in the strain amplitude of the free surface under the action of vibration along this boundary at frequencies close to the expected resonance (natural) frequencies were observed. Thus, it may be concluded that the phenomenon of standing surface-wave formation in fluid volumes under the action of vibration in zero gravity has not been studied thoroughly enough.

The numerical investigation of the action of the axial vibration in zero gravity on a liquid column bounded by solid face surfaces was carried out by us under the "Science–NASA" program within the framework of an agreement between NASA and the Russian Space Agency (project TM-7) [10, 11]. The calculations were made under the action of one frequency from a wide range of frequencies, under the simultaneous action of two different frequencies, and under the impact action on the liquid column. It was found that both one and several lengths of the standing wave could be present on the free surface. Oscillations of the standing wave could be executed at a frequency equal to the applied vibration frequency or smaller than the latter. Under the action of two frequencies, the low frequency was determining and the high frequency distorted the wave form, creating a "ripple." The data obtained by us in numerical calculations enabled us to proceed to the construction of a theory of standing waves in zero gravity.

Formulation of the Problem of Numerical Investigation. To understand the results of numerical calculations and construct a theory of surface waves in zero gravity, we present the mathematical model for numerical investigation, which was described in the reports on project TM-7 and briefly expounded in [10, 11].

Let us consider a liquid column of radius R and length l bounded at the faces by solid surfaces. It is assumed that the wetting angle of the faces is 90° , i.e., in zero gravity the vibration-unperturbed lateral surface is a smooth cylindrical surface. Under the action of axial vibration of frequency f the liquid column is moving as a unit, i.e., the length L is invariable. To determine the free surface motion, we use the VOF (volume of fluid) method [12]. The point of this method is that to describe the boundary one uses a variable, let us call it G , whose value is equal to unity at any point occupied by the liquid and zero at points where it is absent. The mean value of G determines the fractional volume of the cell occupied by the liquid. Cells with a value of G between zero and unity contain a free surface. The normal direction to the boundary is that in which the value of G changes most rapidly. Since G is a discrete function, its derivatives are found numerically and are used to define the gas–liquid boundary and the normal to the boundary. The time dependence of the value of G is defined by an equation in dimensionless variables of the form

$$\partial G/\partial \tau + \text{Re}_\omega [\partial (Gu)/\partial r + \partial (Gv)/\partial z + Gu/r] = 0, \quad (1)$$

to which ordinary Navier–Stokes equations are added in projections on the axes r and z for a viscous incompressible liquid flow in dimensionless form. We write the equations for the coordinate system moving along the z -axis with a vibration frequency:

$$\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial z} = 0, \quad (2)$$

$$\frac{\partial u}{\partial \tau} + \left(u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial r} + \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} \right) + \text{Ar}_r, \quad (3)$$

$$\frac{\partial v}{\partial \tau} + \left(u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} \right) = - \frac{\partial P}{\partial z} + \left(\frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} \right) + \text{Ar}_z + \left(\frac{R}{a} \right) \text{Re}_\omega^2 \sin(\Omega \tau), \quad (4)$$

therefore, in the equation of motion along the z -axis (4) a term with a vibrational acceleration appears.

In writing (1)–(4), as scales for the linear sizes, velocity, time, and pressure, we used the parameters R , v/R , $R^2 v$, and $\rho v^2/R^2$, respectively, and the following boundary conditions: on the solid faces

$$z = 0, \quad z = L, \quad u = 0, \quad v = 0 \quad (\text{liquid adhesion condition}). \quad (5)$$

On the free surface given as a function of height $r = \xi(z)$, the following conditions are fulfilled:
absence of shear stresses

$$\eta \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right) = 0; \quad (6)$$

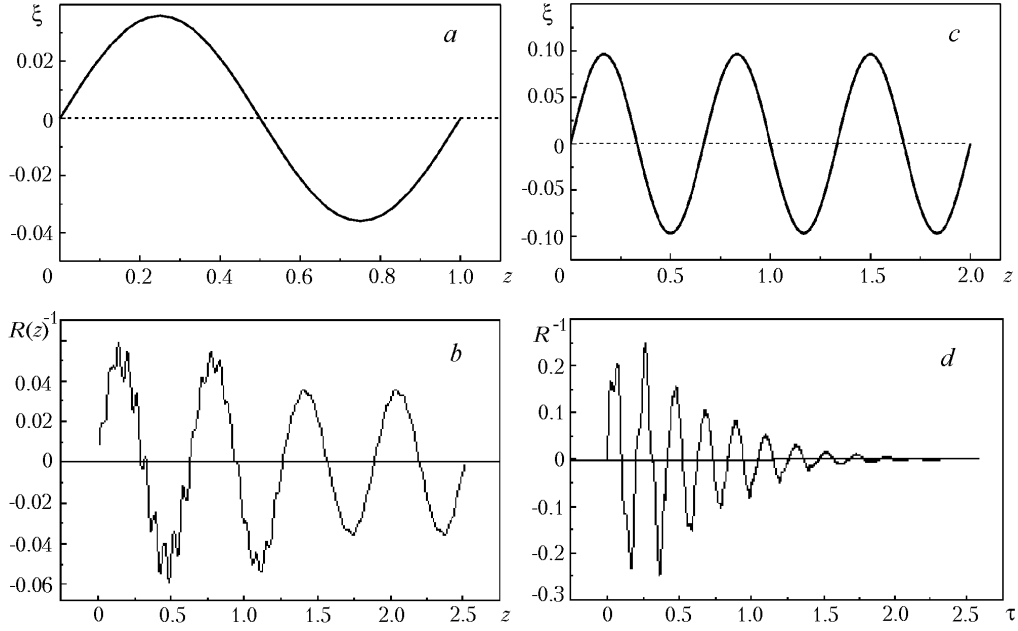


Fig. 1. Profiles of the standing wave (a, c) and the free surface oscillation (b, d) at points $z = L/4$ (b) and $z = L/6$ (d). $K_\sigma = 3260$. $Re_\omega = 10$, $\Omega = 10$ (a); $Re_\omega = 30$, $\Omega = 30$ (c).

condition of equality of normal stresses on a curvilinear boundary

$$P - 2 \left(\frac{\partial u}{\partial r} \right) - K_\sigma^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = 0. \quad (7)$$

For an arbitrary curved line on a plane the following relation is valid:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{(1 - r\xi_{zz} + \xi_z^2)}{(1 + \xi_z^2)^{3/2}}. \quad (8)$$

Here ξ_z and ξ_{zz} are the first and second derivatives with respect to z of a curvilinear surface $\xi(z)$.

The kinematic condition on a deformable free surface is of the form

$$u = \frac{\partial \xi}{\partial \tau}. \quad (9)$$

For approximation of the system of equations (1)–(4), the so-called donor–acceptor method is used [13]. In the calculations, we used a uniform mesh. The calculation algorithm was tested on the known model problems on liquid flow in a pipe with a constant cross section and the liquid column spreading upon inclusion of gravity, taken from [13].

Results of the Numerical Calculations of Surface Waves. Figures 1 and 2a present some of the results of the numerical calculations of the problem on liquid-column free-surface oscillations in zero gravity ($g = 0$) under the action of an axial vibration consisting of one frequency. At vibration parameters $Re_\omega = 10$ and $\Omega = 10$ on the side surface of a liquid column of length $L = 1$ one standing wave is formed (Fig. 1a), which oscillates with an applied vibration frequency $\Omega = 10$ (Fig. 1b). With increasing vibration frequency and rate ($Re_\omega = 30$, $\Omega = 30$) the standing wave at a liquid column of length $L = 2$ has three periods (Fig. 1c) and also oscillates at the vibration frequency (Fig. 1d). Figure 2a shows the liquid oscillation at point $z = L/2$, where the nodal point of a standing wave consisting of one period at the length $L = 2$ is situated. Oscillations occur at a frequency of $\Omega = 50$, whereas the applied vibration

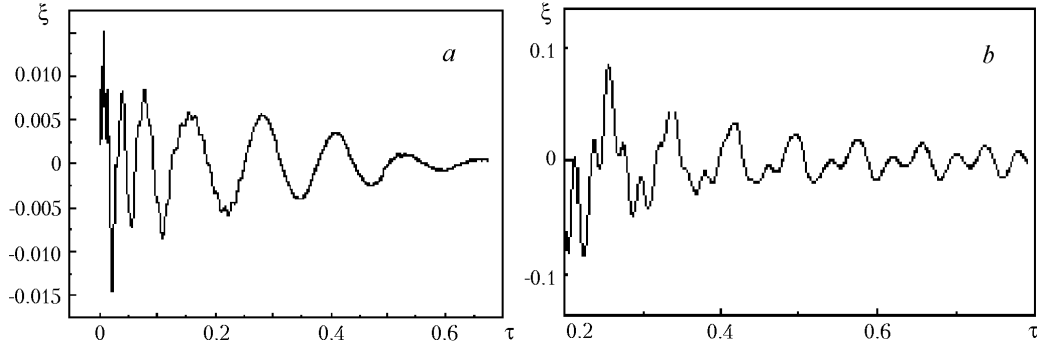


Fig. 2. Free-surface oscillations at point $z = L/2$ (a) [$\text{Re}_{\omega} = 3.12$, $\Omega = 200$, $L = 2$] and at point $z = L/4$ under impact action (b) [$\text{Re}_{\omega 1} = 10$, $\Omega_1 = 300$, $\text{Re}_{\omega 2} = 20$, $\Omega_2 = 480$, $L = 1$].

frequency is equal to 200. Under the action on the liquid column of an impact, whose spectrum consists of two frequencies, first chaotic decaying oscillations of the free surface are observed, which change to oscillations at one frequency corresponding to the natural frequency of the liquid column (Fig. 2b).

New Mathematical Model of Inertial-Capillary Surface Waves. The results of the numerical calculations enabled us to proceed to construct a mathematical model of surface waves excited by vibration in zero gravity. First of all, note that during reciprocating motion of the liquid column as a unit, two forces are acting on it: capillary tension and inertia. The latter can be called a force conventionally, since it cannot be expressed in terms of physical parameters and included in the balance of forces. The fluid motion on the free surface is due to the surface-tension force and is determined in terms of the deformation rate under the action of the capillary forces. For the motion of the free surface to be harmonic throughout the time of action of vibration, two conditions should be met.

According to the first condition, the phase velocity of the standing wave should be not lower than the vibration rate $v_{\sigma} \geq v_{\text{vibr}}$. It can be fulfilled by decreasing the length of this wave, since the phase velocity in the standing capillary wave increases with increasing length of the wave. This means that a free surface of length l should contain as many wave periods as are needed for this condition to be fulfilled.

The second condition is associated with the notion of natural frequency of the liquid column f_e . If the vibration frequency is larger than the natural frequency of the liquid column, then the standing wave oscillates with a frequency lower than the vibration frequency an integer number of times, i.e., $f_e = f_{\text{vibr}}/m$, where $m = 1, 2, 3, \dots$. This phenomenon is depicted in Fig. 2a. The phase capillary velocity $v_{\sigma} \sim (\sigma/\rho R)^{1/2}$ [14]. Here R is the characteristic size for the surface wave. The proportionality coefficient is determined by the geometry, and, for example, in the case of a plane one-dimensional wave it is equal to $(2\pi)^{1/2}$. Knowing the phase velocity and the length of the free surface, one can calculate the natural frequency of the fluid volume.

Using the calculated data on decaying oscillations of the free surface under an impact (Fig. 2b) and the above functional relation for v_{σ} , we obtained the expression for the dimensionless natural cyclic frequency depending on the liquid column length at $n = 1$:

$$\Omega_e = 2.732 K_{\sigma}^{1/2} L^{-3/2}, \quad (10)$$

which can also be used for calculations with $n \neq 1$. In this case, instead of L we put L/n .

Using (10), the phase velocity in the standing capillary wave can be written in the dimensionless form

$$V_{\sigma} = \frac{2.732}{2\pi} (K_{\sigma} n/L)^{1/2}. \quad (11)$$

The first condition written above in dimensional variables is, in dimensionless form:

$$V_{\sigma} \geq \text{Re}_{\omega}. \quad (12)$$

TABLE 1. Comparison of the Analytical Calculations of the Standing Wave Parameters with the Numerical Calculations and Experiment

Re_ω	Ω_{vibr}	L	n	δ	Ω	n	V_σ	Ω_e	δ	Ω
			Numerical calculation			Analytical calculation				
10	10	1	1	0.0358	10 ($m = 1$)	1	24.83	156	0.03	10 ($m = 1$)
30	30	2	3	0.098	30 ($m = 1$)	3	30.4	286.7	0.0954	30 ($m = 1$)
3.12	2.09	2	1	0.013	50 ($m = 4$)	1	17.6	55.15	0.0126	50 ($m = 4$)
			Experiment [8]							
6.974	2.09	5.66	1	0.03	2.09	1	10.44	11.56	0.0358	2.09 ($m = 1$)

The data of the numerical calculations (Fig. 1a, c) show that the standing wave is well described by a sinusoid. Let us write the equation for the standing wave in dimensionless form at any number of wave periods n on the length L :

$$\xi/\delta = \sin(2\pi zn/L) \cos \Omega \tau. \quad (13)$$

To determine the standing-wave amplitude δ , make the following observation. Oscillations in the standing wave permit visualizing the kinetic-to-potential energy conversion. At the instant the maximum wave amplitude is attained the radius rate of motion of the wave is equal to zero (minimum kinetic energy of such a motion), and the potential energy created by the surface tension is maximum. At the instant the side surface becomes smooth ($\xi(z) = 0$) an inverse ratio of these energies takes place. The energy for sustaining a continuous vibrational process is supplied due to the vibration from the outside. Thus, the establishment of the relation between these energy-balance components will permit determination of the standing-wave amplitude δ .

Let us take the surface potential energy difference per unit of length of the liquid column to be equal to the vibration energy transferred from the faces to the flowing liquid layer d . Let us construct the relation for the general case where the liquid column length has room for n periods of the standing wave:

$$(nS_{1/2}/L - 1) \sigma = 0.7d\rho a^2 \omega^2, \quad (14)$$

where $S_{1/2}$ is a surface created by the rotation of one-half of the standing wave, i.e., by a curve segment $R + \delta \sin(2\pi n z/L)$, about the cylinder axis; 0.7 is the coefficient that appeared upon averaging of the rate over the vibration period.

After the calculation of the surface integral we obtained the relation for determining the standing-wave amplitude:

$$(1 + q^2)^{1/2} [E_{\text{McR}}(\pi; q(1 + q^2)^{-1/2}) - E_{\text{McR}}(0; q(1 + q^2)^{-1/2})] + L(2\pi n)^{-1} [\ln((1 + q^2)^{-1/2} + 1) - \ln((1 + q^2)^{-1/2} - 1)] - 2\pi = 1.4\pi(d/R) \text{Re}_\omega^2 \text{K}_\sigma^{-1}. \quad (15)$$

Since we failed to find suitable tables of the McRobert E-function [15], in calculating the surface area in expression (14), we replace half of the sinusoid by half of an ellipse with a large axis equal to half of the standing wave $L/(2\pi)$ and with a small axis equal to δ . Then

$$E[(1 - (4n\delta/L)^2)^{1/2}; \pi/2] = 1 + 0.7(d/R) \text{Re}_\omega^2 \text{K}_\sigma^{-1}. \quad (16)$$

The unperturbed part of the liquid of radius $(R-d)$ under vibration moves as a solid body. Only in the layer d is the vibration energy transferred to the liquid oscillation. If we estimate this layer thickness using the relation for the boundary layer at the vibrating wall $d/R \approx 5(2/\Omega)^{1/2}$ [16], then the last expression can be written as

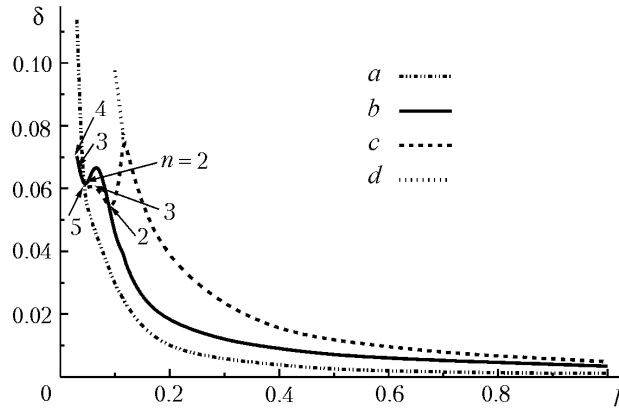


Fig. 3. Standing-wave amplitude in liquid gallium arsenide as a function of the vibration frequency at the following values of the vibrational acceleration: a) $g_{\text{vibr}} = 0.5$; b) 1; c) 2; d) 2 cm/sec^2 , at $n = 1$. $L = 2$, $R = 1 \text{ cm}$.

$$E [(1 - (4n\delta/L)^2)^{1/2}; \pi/2] = 1 + 5 (a/R) \text{Re}_\omega^{3/2} K_\sigma^{-1}. \quad (16a)$$

The argument of the function E includes the sought parameter δ and is determined by the table value of the ordinate calculated by the given parameters in the right-hand side of (16) or (16a).

The relations obtained have restrictions on the side of small vibration frequencies when the thickness of the perturbed liquid layer d becomes larger than the cylinder radius. On the side of large vibrations the approximation is restricted to the use of the incompressible fluid model. Real vibrations existing onboard spacecraft or vibrations that can be created artificially for controlling the liquid flow are within these limits. The calculations by relation (16a) show that when the number of periods of standing waves on the liquid-column length increases, the wave amplitude decreases in the same proportion. Proceeding from the acting effects, the standing waves under consideration can be called inertial-capillary surface waves.

Table 1 compares the data of the numerical calculations and the experiment described in [8] with the calculations carried out using the constructed mathematical model and the approximate relation (16a). It is seen that there is a good agreement in the region of the above-mentioned restrictions. Even in the region of very low frequencies (numerical calculation with $\Omega = 10$, $\text{Re}_\omega = 10$ and experiment [8] with $\Omega = 3.14$) for which the above condition of model limitation is not fulfilled, analytical calculations yield good results. In these cases, we assumed $d = R$ and used relation (16). The most encouraging fact is that the proposed mathematical theory of inertial-capillary standing waves correctly gives the number of periods of the standing wave and the vibration frequency in the case where this frequency differs from the applied oscillation frequency. The proposed mathematical model has no room for the resonance growth of the wave amplitude near the natural frequency of the liquid column. This fact was noted in the experiment performed on the release tower in [9].

Figure 3 gives the results of the calculation by the proposed mathematical model of the standing-wave amplitude for a gallium arsenide (GaAs) liquid zone of radius $R = 1 \text{ cm}$ and length $L = 2$. The frequency dependences of the wave amplitude for three fixed vibrational accelerations are given. For the vibrational acceleration $g = 0.5 \text{ cm/sec}^2$, on the free surface a standing wave consisting of one period ($n = 1$) can always be present. For the other two accelerations in the region of low frequencies the number n can also be greater than one. These places are marked with arrows. In the region where $n > 1$, the curve for $g_{\text{vibr}} = 2 \text{ cm/sec}^2$ is extended by dots to show what value δ would have at $n = 1$.

Heat-and-Mass Transfer under Crystallization of Liquid by the Floating-Zone Method. It is tempting to estimate the action of liquid-column free-surface oscillations on the thermocapillary convection stability and the crystal growth by the floating-zone method in zero gravity. To do this, it is necessary to add to Eqs. (2)–(7) the heat and impurity transfer equations

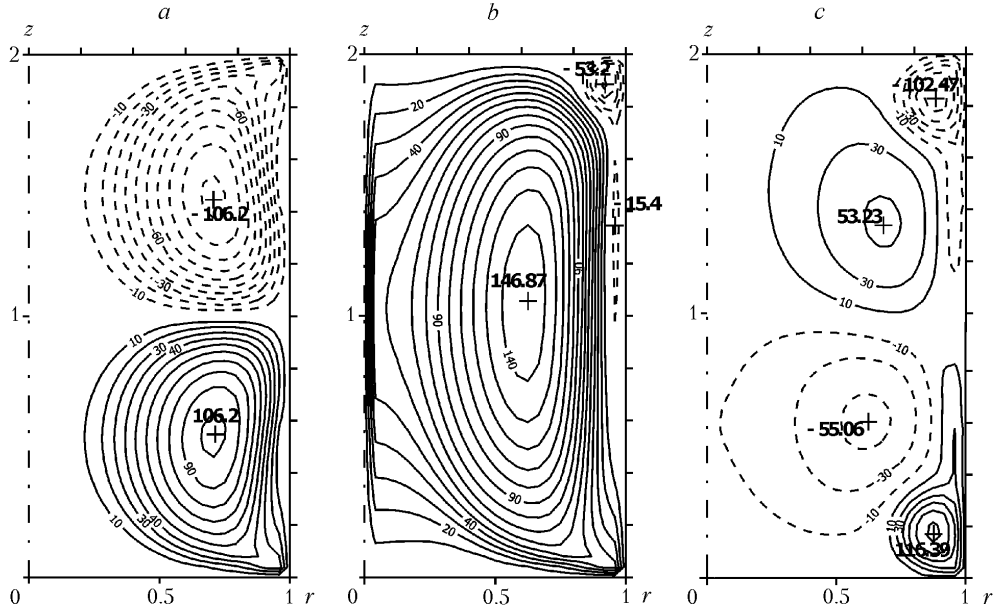


Fig. 4. Structure of the stream (isolines of the stream function ψ) without vibration (a) and with vibration (b, c). $Ma = 1265$, $Pr = 0.023$; $\Omega = 464.1$ ($f = 0.26$ Hz), $\delta = 0.0097$ (b); $\Omega = 392.7$ ($f = 0.22$ Hz), $\delta = 0.012$ (c).

$$\frac{\partial \theta}{\partial \tau} + \left(u \frac{\partial \theta}{\partial r} + v \frac{\partial \theta}{\partial z} \right) = Pr^{-1} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{\partial^2 \theta}{\partial z^2} \right), \quad (17)$$

$$\frac{\partial C}{\partial \tau} + \left(u \frac{\partial C}{\partial r} + v \frac{\partial C}{\partial z} \right) = Sc^{-1} \left(\frac{\partial^2 C}{\partial r^2} + \frac{\partial^2 C}{\partial z^2} \right). \quad (18)$$

Equations (3) and (4) neglect the static lift associated with the continuously acting acceleration of gravity (since $g = 0$ is assumed) and the vibrational convection.

The boundary conditions for them are as follows:

$z = 0$ (crystallization boundary):

$$u = v = 0, \quad \theta = 0, \quad - \left(\frac{\partial C}{\partial z} \right)_s = Re_{cr} Sc (1 - kC_s); \quad (19)$$

$z = L$ (melting original crystal):

$$u = v = 0, \quad \theta = 0, \quad C = 1; \quad (20)$$

$r = 0$ (symmetry axis): the first derivatives of all parameters are equal to zero;

$r = \xi(z)$ is an oscillating free boundary, $u = 0$, $v(z, \tau)$ is given in accordance with the solution of (15) or (16), (16a)

$$\left(\frac{\partial C}{\partial z} \right) = 0, \quad \frac{\partial \theta}{\partial r} = \left(\frac{\partial \theta}{\partial r} \right)_{r=R} \cos \alpha, \quad (21)$$

where the heat flow from the outside to the liquid unperturbed zone is determined as an exponent with a maximum in the cross section $z = L/2$:

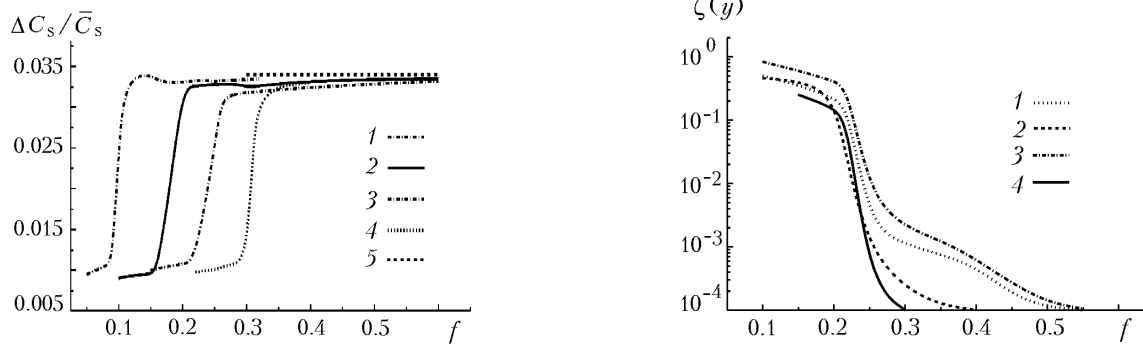


Fig. 5. Relative radial segregation of the phosphor impurity in silicon as a function of the vibration frequency at the following values of the vibrational acceleration: 1) $g_{\text{vibr}} = 0.5$; 2) 1; 3) 1.5; 4) 2; 5) 0 cm/sec^2 . $\text{Ma} = 1265$, $\text{Pr} = 0.023$, $\text{Sc} = 5$, $k = 0.35$, $\text{Re}_{\text{cr}} = 0.1$, $R = 1$ cm, $L = 2$, $A = 0.8$, $B = 0.3$. f , Hz.

Fig. 6. Relative radial segregation $\Delta C_s/\bar{C}_s$ (1, 2) and relative oscillation amplitude of temperature θ (0.5; 0.02) (3, 4) of the phosphor impurity in silicon as a function of the vibration frequency at two values of the Marangoni number: 1, 4) $\text{Ma} = 1265$; 2, 3) 505. $\text{Pr} = 0.023$, $\text{Sc} = 5$, $k = 0.35$, $\text{Re}_{\text{cr}} = 0.1$, $R = 1$ cm, $L = 2$, $A = 0.8$, $B = 0.3$, $g_{\text{vibr}} = 1.5$ cm/sec^2 . f , Hz.

$$\left(\frac{\partial\theta}{\partial r}\right)_{r=R} = A \exp\left[-B\left(z - \frac{L}{2}\right)^2\right]. \quad (22)$$

This function well imitates the boundary condition in the floating-zone method.

The investigation of the influence of free surface oscillations on the liquid flow and the heat-and-mass transfer was begun by studying the thermocapillary convection stability. As one can judge from the publications, such a problem was not considered earlier. There are works on the influence of a stationary curvilinear surface on the thermocapillary convection [17, 18]. The flow in the vicinity of a free surface due to the surface-tension gradient is an example of a shear flow which, by definition, is absolutely stable. In closed fluid volumes having not only a free boundary but also a solid one, such an absolute stability is impossible. It is even more interesting to know how such a flow will be influenced by a perturbation in the form of a variable velocity acting normally to the free surface.

The structure of the flow (current function isolines) without free surface oscillations and with such oscillations at various values of the oscillation parameters are shown in Fig. 4 at one and the same Marangoni number. The structure (Fig. 4b) corresponds to the vibration level at which there are no parametric oscillations in the liquid, although the character of the flow has changed considerably compared to the flow without vibration (Fig. 4a). A small change in the vibration frequency at the same vibrational acceleration leads again to a change in the structure of the flow (Fig. 4c). The instantaneous structure of the flow at a maximum of the free surface deformation rate when this surface is flat is shown. All parameters (velocity, temperature, concentration) oscillate. The change from the stationary regime of convection is very sharp, which is seen in Figs. 5 and 6. Figure 5 shows the change in the relative radial nonuniformity of the phosphor impurity distribution in silicon depending on the vibration frequency at several fixed vibrational accelerations. In the region of large vibration frequencies, this parameter of the impurity-distribution nonuniformity is almost the same as without vibration (upper horizontal straight line). In the transition region, the radial nonuniformity sharply decreases, but simultaneously strong oscillations arise. In Fig. 5, on the left of the transition boundary, the time-averaged values of the parameter $\Delta C_s/\bar{C}_s$ are shown. The relative oscillation amplitudes of the concentration parameter and temperature at a point near the crystallization boundary ($z = 0.02$, $R = 0.5$) are shown in Fig. 6. The vibration regime at which the relative oscillation amplitude reaches 1% is determined by us as a boundary of the change to the oscillation regime of convection. The dependence of the vibration frequency corresponding to this boundary on the vibrational acceleration is shown in Fig. 7 for silicon at fixed sizes of the liquid zone and the Ma-

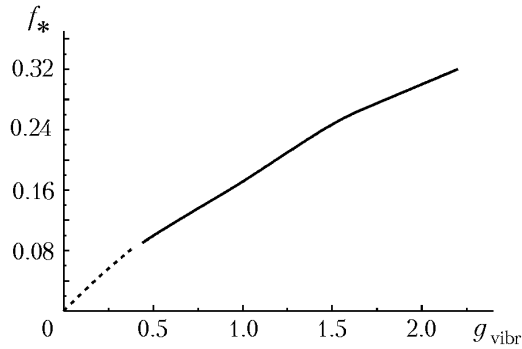


Fig. 7. Vibration frequency at the boundary of transition to the oscillation regime of convection as a function of the vibrational acceleration for silicon. $Ma = 1265$; $L = 2$, $R = 1$ cm, $A = 0.8$, $B = 0.3$. f_* , Hz; g_{vibr} , cm/sec^2 .

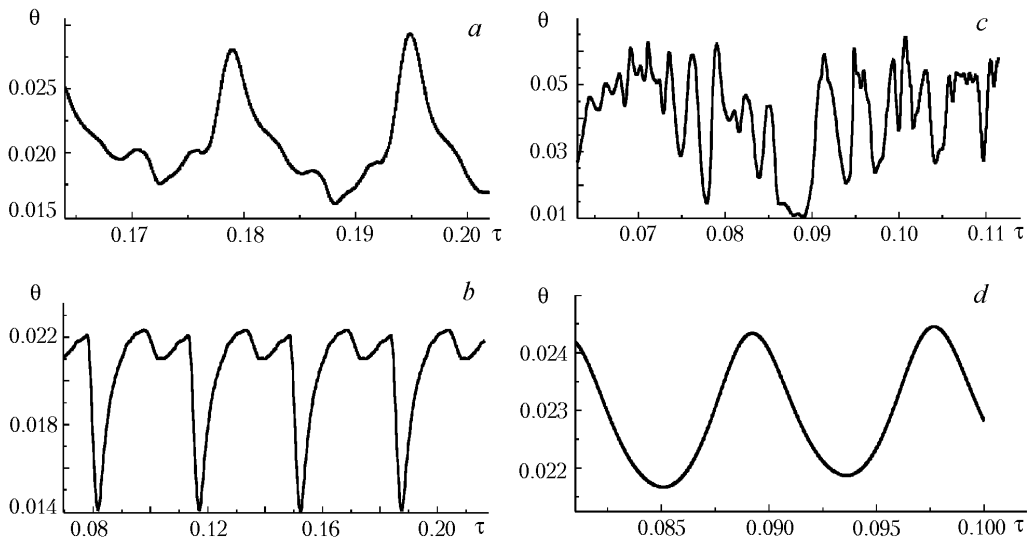


Fig. 8. Temperature oscillations in the liquid near the crystallization boundary ($r = 0.5$, $z = 0.02$) in the regime of oscillation convection [a–c) with vibration; d) without vibration] at various Marangoni numbers: a) $Ma = 505$ ($\Omega = 392.7$, $f = 0.22$ Hz, $\delta = 0.012$); b) 860 ($R = 1$ cm, $f = 0.1$ Hz, $g_{\text{vibr}} = 1$ cm/sec^2); c) 1265 ($R = 1$ cm, $\Omega = 267.8$, $f = 0.15$ Hz, $\delta = 0.018$, $g_{\text{vibr}} = 1.5$ cm/sec^2); d) 2200 ($L = 2$). $Pr = 0.023$, $A = 0.8$, $B = 0.3$.

rangoni number. In this case, the transition boundary very weakly depends on the thermocapillary convection intensity. For instance, for the vibrational acceleration equal to 1.5 cm/sec^2 , $R = 1$ cm and $L = 2$, when the Marangoni number is changed from 500 to 2000, the vibration frequency at which the transition to oscillations is made changes linearly from 0.23 to 0.24 Hz. Note that all parameters of the vibration (frequency, amplitude, acceleration) and standing-wave amplitude for which the calculations have been performed are related by the analytical solution.

Figure 8 gives examples of temperature oscillations in the vicinity of the crystallization boundary at various combinations of dynamic parameters. It should be noted that besides the applied vibration frequency other frequencies, the number of which can be very large, are also present in the spectra (Fig. 8c). If the thermocapillary convection stability is disturbed because of the large fluid velocity at large values of the Marangoni number in the absence of vibration, then the oscillations near the stability boundary are harmonic (Fig. 8d). In this case, the oscillations of temperature and other parameters sharply differ in amplitude and spectra from the above examples when stability is lost due to the action of the surface standing waves. This is clearly seen from comparison of the data given in this figure.

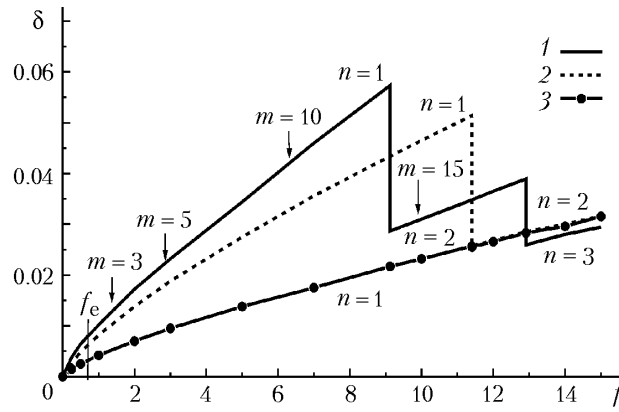


Fig. 9. Standing-wave amplitude in the silicon oil liquid column as a function of the vibration frequency at three values of the vibration amplitude: 1) $a = 0.25$; 2) 0.2; 3) 0.1 mm. $R = 1$ cm; $L = 2$ m, $\nu = 0.1$ cm²/sec. f , Hz.

Discussion. Based on the data of the numerical calculations, we have constructed a mathematical model of standing inertial-capillary waves formed on the free surface of a liquid cylindrical column under the action of axial vibration when the liquid column is moving as a unit. This type of surface waves differs from the well-known capillary waves which are excited when a solid wall placed in a liquid oscillates. The inertial-capillary waves have some analogy with the Faraday waves revealed by experiment in 1831. The Faraday waves also oscillate with a frequency twice as low as the applied vibration frequency. Proceeding from the mechanism of the appearance of Faraday waves, they can be named, by analogy with the term inertial-capillary waves introduced by us, the inertial-gravitational waves.

The calculations performed have shown that vibration can strongly influence the flow and the heat-and-mass exchange in growing crystals by the floating-zone method at certain vibration parameters and physical properties of the liquid. The semiconductor materials usually used in investigations on material science on space vehicles have large values of the parameter K_{σ} . For example, at $R = 1$ cm for silicon $K_{\sigma} = 2.3 \cdot 10^7$ and for gallium arsenide $K_{\sigma} = 3.9 \cdot 10^6$. For such liquids, vibrations existing onboard space vehicles in the normal regime of flight create on the liquid surface standing waves with very small amplitudes. In this case, the standing waves should not substantially influence the process of crystal growth. Of course, this conclusion assumes that crystals are grown in extreme situations: dynamic operations with the vehicle, intensive activity of the crew near the experimental facility, etc. However, if experiments are performed with materials (optical, biological, e.g., aqueous solutions of proteins, etc.) having much smaller values of the parameter K_{σ} , then in this case the existing vibrations can be dangerous.

The small values of the parameter K_{σ} for some liquids (water, glycerol and its water solutions, silicon oils) make them very sensitive to vibrations. These liquids can be used for measuring vibrational accelerations onboard space vehicles, especially in the range of frequencies smaller than 0.1 Hz. The existing accelerometers have a small accuracy in this frequency range. Figure 9 shows the dependences of standing-wave amplitudes on the vibration frequency for a silicon oil liquid column ($\nu = 1 \cdot 10^{-5}$ m²/sec, $\sigma = 0.02$ H/m, $\rho = 998$ kg/m³) for three vibration amplitudes. Arrows point to points where the standing-wave oscillation frequency changes (coefficient m). The kinks on the curves correspond to the points where the standing-wave amplitude changes with changing number of wave periods (change from $n = 1$ to $n = 2$, and so on). In this example, the vibration frequencies and accelerations used by us for the calculation are close to those which exist on the International Space Station (ISS).

Using relation (16a), we can obtain an explicit relation between the standing-wave amplitude and the vibrational acceleration:

$$g_{\text{vibr}} = 0.447 R \omega^2 K_{\sigma}^{1/2} \Omega^{-3/4} [E(\epsilon) - 1]^2. \quad (23)$$

Vibrations on board are characterized by components on all three axes (although two of them dominate), and the deformation of the liquid column under such conditions has a complex spatial form. The proposed method is not suitable for such a situation. However, using a simple mechanical device, a vibrosuspension [19], one can reduce the whole set of vibration frequencies to one equivalent frequency, which can be measured by means of the liquid column

and the above mathematical model. The results of the investigation presented show that the device called in [19] vibroprotective is not such, since it creates a higher vibration at one, and low frequency, which increases the negative effect of vibration on the crystal growth.

The experiment on the action of vibration on the liquid column was included in 1998 in the program of works on the Russian module of the ISS.

Conclusions. An analytical model of inertial-capillary waves arising on the free surface of a liquid column under the action of vibration in zero gravity has been developed. The model has been tested by comparison with the data of numerical calculations and the experiment performed in microgravity. Analysis shows that using this mathematical model and a special vibrosuspension, one can measure the vibrational acceleration equivalent to the acting set of vibration frequencies existing on space vehicles, including the ISS.

A program for numerical calculation of the heat-and-mass exchange in growing crystals by the floating-zone method in zero gravity with regard for the standing surface waves has been developed. The boundary of transition to the oscillating regime of thermocapillary convection and its dependence on the vibration frequency and vibrational acceleration has been determined. A weak dependence of the transition boundary on the Marangoni number has been shown. The program also permits calculations for the case where surface waves arise under crystal vibrations with a small amplitude.

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NOTATION

A, B , dimensionless coefficients in (22); a , vibration amplitude, m; $Ar = gR^3/v^2$, Archimedes number; c , impurity concentration in liquid, kg/m^3 ; $C = c/c_0$, dimensionless concentration of impurity; D , diffusion coefficient of impurity in liquid, m^2/sec ; d , thickness of the vibration-perturbed liquid layer, m; $E_{\text{McR}}(\pi; \gamma)$, MacRobert function, dimensionless; $E(x; \pi/2)$, complete integral of the second kind, dimensionless; f , frequency, Hz; g , gravitational acceleration, m/sec^2 ; G , portion of liquid in the calculation cell; l , liquid column length, m; k , equilibrium impurity distribution coefficient; $K_{\sigma} = \sigma R \rho^{-1} v^{-2}$, dimensionless capillary constant; $L = l/R$, dimensionless liquid column length; $Ma = -(\partial\sigma/\partial T)R\Delta T/\rho v \chi$, Marangoni number; m , ratio of the vibration frequency to the oscillation frequency of the standing wave; n , number of standing wave periods at length L ; p , pressure, Pa; $P = pR^2/\rho v^2$, dimensionless pressure; $Pr = \nu/\chi$, Prandtl number; $q = 2\pi n\sigma/L$, parameter in (15); R , radius of unperturbed liquid column, m; R_1, R_2 , radii of principal curvature, m; r, z , cylindrical coordinates, m; $Re_{\text{cr}} = v_{\text{cr}}R/v$, dimensionless crystal growth rate; $Re_{\omega} = \alpha\omega R/v$, dimensionless vibration rate; $Sc = \nu/D$, Schmidt number; x , parameter in the definition of the independent variable of the complete elliptical integral of the second kind; T , temperature, K; t , time, sec; u and v, r and z , components of velocity, respectively, m/sec; $V_{\sigma} = 2.732/2\pi(K_{\sigma}n/L)^{1/2}$, dimensionless capillary velocity; α , angle between normals to a point on the perturbed free surface and to the surface of $r = R$; $\Delta T = T_{\text{max}} - T_0$, characteristic temperature difference in the system, K; $\Delta C_s = C_{s,\text{max}} - C_{s,\text{min}}$, radial concentration difference in the liquid; η , dynamic viscosity, $\text{kg}/(\text{m}\cdot\text{sec})$; ν , kinematic viscosity, m^2/sec ; ξ , deviation of the free surface from the unperturbed state, m; ρ , density, kg/m^3 ; σ , surface tension, N/m; χ , thermal diffusivity, m^2/sec ; ω , circular frequency, sec^{-1} ; $\Omega = \omega R^2 v^{-1}$, dimensionless circular frequency; $\varepsilon = (1 - (4n\delta/L)^2)^{1/2}$, parameter in relation (23); $\gamma = q(1 + q^2)^{-1/2}$, parameter in the argument of the MacRobert E-function; δ , standing-wave amplitude; $\zeta(y)$, relative oscillation amplitude of parameter y ; $\theta = (T - T_0)/\Delta T$, dimensionless temperature; $\tau = t\nu/R^2$, dimensionless time; ψ , dimensionless stream function. Subscripts: 0, initial value; cr, crystallization; e, eigenvalue; max and min, maximum and minimum values; s, at the crystallization boundary; σ , pertaining to the capillary effect; *, at the boundary of transition to oscillations; bar, averaging over the crystallization boundary; vibr, vibration.

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